Astrophysical Black Hole horizons in a cosmological context: Nature and possible consequences on Hawking Radiation

I. INTRODUCTION

This paper is about the location of apparent horizons when astrophysical black holes form from the collapse of a single astrophysical object in a cosmological context. A current view with many proponents is that when black hole formation takes place, a singularity forms and is hidden by an event horizon, but then the blackhole completely evaporates away within a finite time due to Hawking radiation [7, 33, 37]. The outcome however has recently been challenged by various people proposing that the black hole will not evaporate away [1, 21, 29, 62], while others propose, using different arguments, that a global event horizon will never form [3, 10].

A key issue in this debate is the location of local apparent horizons, characterised as marginally outer trapped 3-surfaces (MOTS), which are arguably crucial to Hawking radiation emission. MOTS are important features of the geometry of black holes, inter alia leading to the prediction of the existence of singularities in the classical black hole case [36, 56]. They are distinct from the global event horizon in realistic dynamical cases, when a pair of MOTS surfaces form (an inner one - the IMOTS and an outer - the OMOTS) as the radius of the collapsing object falls in past the critical radius \( r = 2M \). They will be affected by infalling cosmic background radiation (CBR), as well as by backreaction from ingoing and outgoing Hawking radiation; and these effects must be taken into account in realistic models of black hole radiation.

In this paper we will show under what conditions each of these surfaces is timelike or spacelike, and where they are located. We find that the effect of the CBR on the nature of the outer MOTS surface may be a crucial feature in determining the context of Hawking radiation. Also, in realistic astrophysical contexts, the inner surface does not lie where Bardeen places it in his recent paper on black hole existence [3]; this may significantly affect the outcomes he claims.

A further paper will explore the implications for emission of Hawking radiation and black hole evaporation. This paper only considers spherically symmetric collapse; the situation will be much more complex in the case of rotating black holes, but the spherical case indicates the kind of outcomes we may expect in that case also.

A. The context

This section sets the general context within which the issue of the location of the event horizon arises. We are concerned with the case of astrophysical black holes situated in a realistic cosmological context. There are then three forks in the possibilities that will determine the outcome of Hawking radiation emission processes. The first two possibilities motivate the further studies in this paper.

1. The cosmic context

Considering black hole formation in a cosmological context, two features need to be taken into account, based on our present understanding of the universe.

Cosmic Background Radiation. Since the time of decoupling, the universe is everywhere pervaded by cosmic background radiation (CBR), initially at a temperature of 4000K but then cooling down to the present 2.73K, when the radiation is microwave background radiation (CMB) [22, 57]. Consequently any black hole formation will be subject to this infalling radiation [67]. This will affect the nature of the OMOTS surface, as discussed below.

Cosmic constant. The late universe is dominated by dark energy that is consistent with existence of a small positive cosmological constant. As the cosmological constant does not decay away, the late time evolution of the universe is de Sitter, and future infinity is spacelike [36, 55].

This means that, unlike the asymptotically flat case, there is no single event horizon for any observer outside a black hole. Different cosmological event horizons (which are in the far future) occur for different observer motions that end up at different places \( p_i \) on future infinity (see Figure 4 below), because the event horizon for an observer ending up at \( p_i \) is, by definition, the past light cone of \( p_i \) [36, 55].
2. **Key question 1: Local of global?**

The first key question is whether the location of Hawking radiation emission is locally or globally determined. Is it just outside a globally determined event horizon, as argued strongly by Don Page (private communication), or just outside a local horizon (a MOTS surface) that is locally determined, as argued strongly by Visser [66] and supported by others [11, 15, 52]? In fact the source of the Hawking radiation is still a grey area and we need to perform very detailed and deep semiclassical investigations on the global aspects of physically realistic gravitational collapse to pinpoint such a source. The attempts so far, to introduce a timelike emitting surface outside the event horizon (for example [3, 38, 39, 63]) are ad-hoc in the sense that the existence of such a surface lacks a proper geometrical interpretation. In the last two works mentioned above, this emitting surface has zero energy density but non-zero surface stress which is unphysical. It is hard to motivate why such a surface should exist in an absolutely regular and future asymptotically simple spacetime outside the black hole. On the other hand trapped regions (which have proper geometrical interpretation in terms of null congruences) do play an important role in the “particle pair-creation picture” of Hawking radiation. Hence for a local analysis it is quite natural to assume that the boundary of the trapped region is the source of the radiation. In the case of an unperturbed Schwarzschild black hole, since this boundary exactly coincides with the future event horizon, the event horizon and its vicinity can be considered as a global source. However the problem arises when this degeneracy is broken by infalling matter in the black hole. The viewpoint underlying the basis of this paper is that the Hawking radiation emission must be locally, rather than globally, determined. This is for two reasons. Firstly, this seems to be the only way the concept of a blackhole locally emitting radiation makes sense (it seems absurd that we have to wait until the end of the universe before we know what happens locally [66]). Secondly, because of the point just made: in a realistic context, future infinity outside the black hole is spacelike rather than null, so different observers who are initially near the black hole will experience different event horizons. Which one should we associate with the radiation? The prescription is not well defined.

3. **Key question 2: Timelike or spacelike?**

Assuming that Hawking radiation emission is associated with a MOTS surface, the question then is, does it matter whether this surface is timelike, spacelike, or null? If the calculation is based on the idea of tunnelling (e.g. [52]) then it will be applicable only if the MOTS surface is timelike or null (when “inside” and “outside” can be defined). If it is spacelike, then a tunnelling viewpoint is simply not applicable (spacetime regions will be “before” or “after” the surface, but not inside or outside it). The same applies to heuristic explanations based on one of a pair of virtual particles being trapped behind the horizon (they cannot be trapped behind a spacelike surface) or using S-matrix ideas (scattering takes place off a timelike world tube, not a spacelike surface). These mechanisms can only work if the horizon is timelike.

Now the particle picture is considered suspect by many workers. As emphasized by Paul Davies and Malcolm Perry (private communication) what is needed to make this conclusive is to calculate the stress energy tensor associated with local horizons (see e.g. [7, 11]), and see if this confirms what is suggested by the particle picture or not. That will be the subject of a separate paper.

For the present paper, the issue is that whether a MOTS surface is timelike or spacelike is not only geometrically important, because it determines the relation of the MOTS surface to global event horizons, but it may also play a key role in the Hawking radiation emission.

4. **Key question 3: vacuum or fluid?**

The above two questions assume that the emission of radiation due to quantum processes in a collapsing black hole context is due to the properties of the vacuum domain near the event horizon or trapping surface. However there is another effect at work: the interior of the collapsing fluid is a time-varying gravitational field due to the collapse process, and this can potentially lead to particle creation in an evolving fluid filled spacetime, as noted long ago by Leonard Parker [53]. This is the mechanism of particle creation by a fluid collapsing as it forms a black hole that is discussed by Hawking ([54]: pp 207-208) and Birrell and Davies ([7]: pp 250-262). In this case the role of the horizon is not creation of the radiation, rather it is modulation of the propagation of the radiation that has already been emitted in the time-dependent gravitational field of the fluid; the result is that the outcome is independent of the details of the fluid collapse [7, 53].

This is also the mechanism considered by Mersini-Houghton in her recent paper [45], based on the Hartle-Hawking vacuum, and the assumption that the energy momentum tensor of the ingoing Hawking radiation is that of radiation
in thermal equilibrium. However the Hartle-Hawking vacuum is based on the unphysical case of a white hole being in thermal equilibrium with the black hole; also the ingoing radiation may have a form different from thermal equilibrium. Thus it is still an open question as to how significant this mechanism is compared to the horizon based mechanisms usually associated with Hawking radiation \cite{34}.

What we need to do to convincingly determine the outcome of this fluid based mechanism is to study the evolution of a collapsing body, and then ask what difference do quantum fields make to this dynamical situation. It will be significant to see how this relates to the nature and location of global and local horizons that are discussed in this paper.

\section*{B. The dynamical horizons}

There is a marginal outer trapped 3-surface (the OMOTS) that lies outside the outgoing initial null surface generated by the initial marginally trapped 2-surface (the $S_{MOTS}$) that marks the onset of black hole dynamics. The OMOTS surface is locally determined; it moves outwards with time because of incoming Cosmic Microwave Background radiation, so that the associated mass $m_{out}(u)$ increases with time, and the surface $r = 2M_{out}(u)$ is spacelike. Because the OMOTS bounds the trapped domain in spacetime (the outward directed null geodesics are converging inside this surface), it determines the outside edges of any future singularity that may occur. Thus it non-locally determines the location of the event horizon. OMOTS is a spacelike dynamical horizon whose properties characterise the exterior mass $m_{out}$ and angular momentum $J_{out}$ of the body. However its nature could possibly be changed to timelike at late times by the back reaction of Hawking radiation on its geometry. That is one of the important issue to be investigated.

We make the case below that there is an inner timelike marginally outer trapped 3-surface (the IMOTS), which is a dynamical horizon \cite{2}. This surface lies inside both the OMOTS surface and the event horizon and it’s location is locally determined. It is timelike and also emanates from the $S_{MOTS}$ 2-surface, which is therefore a bifurcation surface originating both MOTS surfaces. It is potentially possible that backreaction from Hawking radiation causes them to merge again in the future at a final $F_{MOTS}$ 2-surface, if the OMOTS surface eventually becomes timelike. Whether that happens or not depends on detailed balance between local dynamics of the fluid and the incoming Hawking radiation in those cases where the OMOTS is timelike at late times.

\section*{C. This Work}

The paper considers these issues in the case of a single black hole with spherical symmetry, where the relevant exterior solution is the exterior Schwarzschild solution surrounding the single collapsing mass. The result is probably stable in the case of more general geometries such as rotating black holes, perturbed black holes, and if there are later infalling shells of matter. The paper does not consider multiple black holes or black hole collisions. Since we consider only astrophysically relevant situations, it also does not consider charged black holes either.

As we confine our attention to spherically symmetric black holes produced by the gravitational collapse of a massive star, the Schwarzschild solution is the basic relevant exterior metric but the interior will be modelled by spherical fluid body: it might be a Friedmann-Lemaître-Robertson-Walker (FLRW) metric, a Lemaître-Tolman-Bondi (LTB) metric, or a spherical metric with pressure \cite{22}. However in addition there may be incoming or outgoing radiation in both the exterior and interior regions; we therefore need to consider generic spherically symmetric metrics. For technical reasons it is convenient to consider a class of spacetimes which are a generalisation of spherically symmetric metrics: namely \textit{Locally Rotationally Symmetric} (LRS) class II spacetimes \cite{18,61,65}. These are evolving and vorticity free spacetimes with a 1-dimensional isotropy group of spatial rotations at every point. Except for few higher symmetry cases, these spacetimes have locally (at each point) a unique preferred spatial direction that is covariantly defined.

To describe this class of spacetimes in terms of metric components, we use the most general line element for LRS-II can be written as \cite{61}

$$ds^2 = -A^2(t, \chi) dt^2 + B^2(t, \chi) d\chi^2 + C^2(t, \chi) \left[ dy^2 + D^2(y, k) dz^2 \right],$$

where $t$ and $\chi$ are parameters along the integral curves of the timelike vector field $u^a = A^{-1} \delta_0^a$ and the preferred spacelike vector field $e^a = B^{-1} \delta_0^a$. The function $D(y, k) = \sin y, y, \sinh y$ for $k = (1, 0, -1)$ respectively. The 2-metric $dy^2 + D^2(y, k) dz^2$ describes spherical, flat, or open homogeneous and isotropic 2-surfaces for $k = (1, 0, -1)$.

Spherically symmetric spacetimes are the $k = 1$ subclass of LRS-II spacetimes.

We can easily see that the physically interesting spherically symmetric spacetimes (for example the Schwarzschild, FLRW, LTB, and Vaidya solutions) fall in the class LRS-II. Hence both the interior and the exterior of a collapsing
star can be described by this class, which can be characterised covariantly through the properties of the vector fields $u^a$ and $e^a$ [63].
